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## ABSTRACT

Multivariable queries can be processed in the data base management system INGRES. The general procedure is to deconpose the query into a sequence of one-variable queries using two processes. One process is reduction which requires breaking off components of the query which are joined to it by a single variable. The other process, tuple-substitution, involves substituting for one of the variables a tuple at a time. The query processing algorithm has been developed for QUEL, the data language for INGRES. Algorithms for reduction and for choosing the variable to be substituted are given. The decision about which variable to substitute depends on estimation of costs, and some procedures for making cost estimates are outlined. (Author/CH)

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Eugene Wong and Karel Youssefi

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## Decompositior - A Strategy for Queny Processing <br> Euger $\mathrm{r}_{\mathrm{e}}$ Worg ard Karel Youssefi

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Abstract

This paper deals. with the strategy for processing multivariable queries in the data base management system INGRES. The general procedure is to decompose the query irto a sequerice of one-variable queries by alterratirg betweer (a) reduction: breaking off comporierts of the query which are joiried to it by a sirgle variable, ard (b) tuple-substitution: substitutirg for ore of the variables a tuple at a time. Algorithms for reduction ard for choosirg the variable to be substituted are given. Ir most cases the latter decision deperds or estimation of costs and heuristic procedures for makirg such estimates are outlined.


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## 1. Introduction

The structural simplicity of a relational data medel encourages the use of a non-procedural data sublarguage which specifies what is to be found rather than how it is to be found. Thus, it is not surprising that rearly every one of the relational languages which have beer proposed is nor-procedural. As is generally true with high level languages, a price which may have to be paid is a loss of efficiency. For a relational data base of any size and for queries spanning several relations, the price can be fearsome. Results of various degrees of generality on improving search strategies for a relational data base system have been reported by Palermo [PALET2], Astrahan and Chamberlin [ASTR75], Rothnie [ROTH74,ROTH75], Pecherer [PECH75], Smith and Chang [SMIT75], and Todd [TODD75]. Nonetheless, the lack of a general approach to optimizing query processing remains a major impediment to achieving a satisfactory degree of efficiency for nor-procedural relational languages.

The purpose of this paper is to describe in some detail the query processing algorithm developed for QUEL ' [HELD75], which is the data language for the INGRES system. Insofar as the problems encountered in QUEL are common to all ron-procedurdl relational languages, "their solution should find general application.

Ir section 2 a brief description of QUEL, the query language to be processed, is presented. In section 3 we sketch a skeletal outline of the decomposition algorithm emphasizirg the functions of the component algorithms and the flow of information and control amons them. The details of the component algorithms are presented in subsequent sections.
2. QUEL

A complete definition, of QUEL is. given 'in [HELD75]. Here, we shall confine ourselves to a brief description sufficient to make the processing strategy comprehersible. There are four commands: RETRIEVE, REPLACE, DELETE, APPEND. Ań update command is turned into a RETRIEVE command which is then followed by a low level tuple-by-tuple operation.' We shail restrict our attention to RETRIEVE. A statement to retrieve in QUEL has the following form.

RANGE OF (Variable [,Variable]) IS
(Relation Name \{,Relation Name])
RETRIEVE [INTO result rame] (Target List)
WHERE Qualificatior
Example 2.1:
Corsider a data base with relations
Supplier (S\#, Sname, City)
Parts (P\#, Pname, Size)
Supply (S\#, P\#, Quantity)
ard a query to find the names of all parts supplied by suppliers in New York. This can be stated in QUEL as follows:

RANGE OF ( S, P, Y, ) IS (Supplier, Parts, Supply)
RETRIEVE INTO NYparts (P.Pname) WHERE (P.P\#= Y.P\#)
AND. (Y.S\#=S.S\#)
AND' (S.City $=$ 'New York')
From the point of view of query processing there are two principal sources of complexity. First, QUEL permits aggregation operators such as MAX ard AVG, and resting of such operators. Secondly, queries involving several variables require deft handling in order to avoid the obvious possibility of combinatorial growth. For example, if the query ir Example. 2.1 is processed by first formirg a cartesiar product, then the number of tuples to be scanned is equal to the product of the gardinalities of the three relations. In our system all aggregations are performed on single relations. If an aggregation is to be done on a subset of the product of several relations, the subset must first be assembled by processing a multivariabe query. Aggregations once evaluated are kept for possible reuse until updates render them obsolete. Ir the remainder of the paper we shfall deal only with aggregation-free queries, and the thrust of the queryprocessing strategy is to cope effectively with
aggregation-free bu't multivariable queries.
Let $X=\left(X_{1}, X_{2}, \ldots, X_{n}\right)$ denote the variables declared in the range statement, and let $R_{1}, R_{2}, \ldots, R_{n}$ be their respective ranges. Then the qualification can be considered to be a Boolean function $B(X)$ on the cartesian product $R=R_{1} \times R_{2} \times \ldots \times R_{n}$. The target list car be considered to fe $a^{n}$ set of functions ( $T_{1}(X)$, $\left.T_{2}(X), \ldots, T_{m}(X)\right)=T(X)$ on the product space, and the result relation of the query is constructed by evaluating - $T(X)$ on the subset of $R$ defined by $B(X)=1$, and eliminating duplicate tuples. We note that for a query free of aggregation operators each tuple $X$ in the product space $R$ contains enough information to completely determine the values of $B(X)$ and $T(X)$.

The interprefation of QUEL statements suggests the following procedure for their processing:
(a) Product: A cartesian product of the range relation is formed.
(b) Restriction: Tuples $X$ in the product which satisfy $B(X)=1$ are determined.
(c) Computation and Projection: $T(X)$ is computed on the subset determined in (b) and duplicate tuples are eliminated.

Unfortunately, this procedure is as-inefficient as it is obvious. The cardinality of the product $R$ (i.e. , the rumber of tuples in $R$ ) is equal to the product of the cardinalities of, $R_{1}, i=1,2, \ldots, n$. It does not take very large relations or very many of them to make this number enormous. Aside from the difficulty of havirg to form and store a very large relation, to determine the subset which satisfies $B(X)=1$ requires examining a number of tuples equal to the cardirality of $R$.

## 3. Decompositior

The query processing stratepy that we have adopted nas two overall objectives:
(a) No cartesian product - The result relation is to be constructed by assembling comparatively small pieces, rather thar by paring down the cartesian product.
(b) No geometric growth - The rumber of tuples to be scanined is to be kept as small as possible, arid for most queries this number is much less thar the cardirality of R.

Our gereral procedure is to reduce ar arbitrary multivariable query to a sequerce of siriple-variable ones. We call this process decomposition. Observe that the first objective is automatically achteved by such ar approach. To attain the second requires a detailed examiration of the tactical moves which are available.

The decision to reduce multivariable queries to one-variable ores separates the overall optimization into two levels. It has obvious, advartages in structurinf the optimization procedure which otherwise may well become urbearably complex. The orily situdtion in which our approach may be undesirable is wher, inter-relational information such as "links" [TSIC75] is availableir which case the desirable atomic urits may be two-variable queries.

It is useful to distinguish two types of operations which are repeatedly invoked ir decompositior.
(I) Tuple substitution: -Ar- ${ }^{\prime} r_{i}^{*}$-variable query $Q$ is replaced by-a family of ( $r_{1}-1$ )-variable queries resulting'from substituting for one of its variables tuple by tuple, i.e.,
$c\left(X_{1}, X_{2}, \ldots, X_{n}\right) \rightarrow\left\{Q_{\alpha}^{\prime}\left(X_{2}^{\prime \prime}, X_{3}, \ldots, X_{r_{2}}\right) ; \alpha \in R_{1}\right\}$
(II) Detachment of a subquery with a single overlapping variable : A query $Q$ is replaced by $Q^{\prime}$ followed by $Q^{\prime \prime}$ such triat $Q^{\prime}$ drd $Q^{\prime \prime}$ nave only a single variable in commor.

Operations of these two types suffice to decompose ary query completely. Indeed, a series of
successive tuple substitutions is sufficient, albeit taritamourt to formirig the cartesian product. Tuple substitution for a sirgle variable means that the cost of processing the remaininf portion of the query is muItiplied"by a factor which in most cases is equal to the cardinality of the rarige of the substituted variable. It is important, therefore, that the parges of the variables be reduced as much as possible before substitution takes place. The most stoyaightforward way of doing this is through restriction ar, projection, which are special cases of (II). Sometning equivalerit to such a step has been proposed in every paper or optimizing query processing.

## Example 3.1

Corsider a data base with three relations
Supplier ( $\mathrm{S} \#$, Sname, City)
Parts (P非, Prame, Size)
Suppiy (S\#, P非, Quaritity)
ard a query $Q$ :


then the successive detachmert of subqueries can be
represented by


Ir this example cperations of type II have reduced 0 to three ore-variable queries $21,02, Q 3$ which car. be processed $i_{1}$ parallel cr in arbitrary order, followed by a 2-variable query 04 , ard ther another $2-v a r i a b l e ~ q u e r y ~ 05 . ~ Q 4 ~ a r d ~ 05 ~ c a r r o t ~ b e ~ f u r t h e r ~, ~$ reduced by operations of type II, ard tuplesubstitution must be used to complete the decompositicr. ide rote, however, the rariges of the variables ir Q4 and OX are likely to be very much smaller thar. the oriniral relatiors, ard tuple substitutior at these stages is relatively harmiess. As an example of tuple substituticr, corisider

Q5 : RETRIEVE (S.Srame) WHERE (Y.S\#=S.S\#)
Suppose that at this poirt the range of $Y$ is the relatior

| SiF |
| :---: |
| 101 |
| 107 |
| 203 |

Ther, suacessive substitutior for Y yields
05(101): RETRIEVE (S.Srame) WHERE (S.S\#=101)
05(107): RETRIEVE (S.Srame) VHERE (S.S非=107)
Q5(203): RETRIEVE (S.Srame) WHERE (S.S\#=203)

We note that unlike SEQUEL [ASTR75], QUEL has ro block structure ard there is ro a priori preferertial crder of variables ir substitution.

The gereral situation covered by (II) is the 'followirg: Corsider a ouery of the form

RANGE OF $\left(X_{1}, X_{2}, \ldots, X_{n_{2}}\right)$ IS $\left(R_{1}, R_{2}, \ldots, R_{n}\right)$
Q
RETRIEVE $T\left(X_{1}, X_{2}, \ldots, X_{m}\right)$
WHERE $B^{n}\left(X_{1}, X_{2}, \ldots, X_{m}\right)$
AND

$$
B^{\prime}\left(x_{m}, x_{m+1}, \ldots, x_{n}\right)
$$

It is natural to break off $B^{\prime}$ to form
RANGE OF ( $\left.X_{m}, X_{m+1}, \ldots, X_{n}\right)$ IS ( $R_{m}, R_{m+1}, \ldots, R_{n_{1}}$ )
Q' RETRIEVE INTO $R_{m}^{\prime}\left(T^{\prime}\left(X_{m}\right)\right)$
WHERE $B^{\prime}\left(x_{m}, x_{m+1}, \ldots, x_{n}\right)$
where $T^{\prime}\left(X_{M}\right)$ cortains the information on $X_{m}$ needed by the remainder of the query which can now be expressed as

RANGE OF $\left(X_{1}, X_{2}, \ldots, X_{m}\right)$ IS $\left(R_{1}, R_{2}, \ldots, R_{m}{ }^{\prime}\right)$
Q" RETRIEVE $T\left(X_{1}, X_{2}, \ldots, X_{m}\right)$
WHERE

$$
B^{\prime \prime}\left(x_{1}, x_{2}, \ldots, x_{m}\right)
$$

Observations: (1) $Q "$ is necessarily simpler than the original query $Q$ since $m \leq r_{\text {a }}$ and $R^{\prime}$ is smaller, than $R_{m}$. Even for the worst possible case where $R^{\prime}=R_{m}$ gnd $m=n, Q^{\prime \prime}$ is no worse than $Q$. (2) The detacfmert ${ }^{\prime}$ of $Q^{\circ}$ does not lead to an increase in the maximum number of variables for which substitution has to be made. To see this, note that the maximum number of variables to be substituted for in an n-variable query is n-1. Herce, this number is $(n-m+1)-1$ for $Q^{\prime}$ and $m-1$ for $Q^{\prime \prime}$ so that the total is again $n-1$. (3) $Q^{\prime}$ and $Q^{\prime \prime}$ are strictly ordered. $Q^{\prime}$ needs no information from $Q^{\prime \prime}$ so that it can be processed completely before processing on $Q^{n}$ begins. At any given time we only need to deal with a total of $n$ or less variables.

Two special cases of one overlappingvariable subqueries are worthy of special note. First, it may happen that the detached subquery $Q^{\prime}$ has no variable in common with the remainder $Q^{\prime \prime}$. That is, $B^{\prime}$ is a function of only ( $X_{m+1}, \lambda_{0}, X_{n}$ ) and not of $X_{m}$. In such a case we stall say $Q^{\circ}$ is a disjoint.subquery. The interpretation of this situation is that if $B$ ' is satisfied by a nonempty set then $Q$ is equivalent to $Q^{\prime \prime}$, otherwise $Q$ is itself void, i.e., its result is empty. The second special case arises when $m=n$ and $B^{\prime}$ is a one-variable query.

This is a frequent and important occurence, as the previous example illustrates. We say a query is corariected if it has ro disjoirit subquery, one-free if it has ro ore-variable subquery, and irreducible if it has no one-overlapping-variable subquery. $\mathrm{Ar}_{\mathrm{a}}$ irreducible query is obviously both cormected ard one-free.

Broadly speakirig, we will always break up a query into irreducible componerts before tuplesubstitution. Ir effect, we will always prefer rot to tuple-substitute if it car be avoided or postporied. Although examples can be corstructed to show that such a choice is rot always optimal, in gereral this is rot a bad heuristic. Detachirg subqueries irvolves an additive growth in complexity, while tuple-substitution incurs a multiplicative growth. Our decompositior algorithm is recursively applied to all the subqueries which are generated.

The Decomposition Algorithm corsists of four sub-algorithms: Reduction, Subquery Sequericirg, Tuple Substitution and Variable Selection and makes use of the Orie-Variable Processor of the system. The interaction among these comporiert processes is irdicated ir Figure 3.1 below



Fisure 3.1 Flow of Coritrol ir Decomposition

The fact that the decomposition algorithm is recursive is made clear by the existence of a sequerce of callirg-paths (Reductior-Subquery Sequercirp,Tuple Substitution-Reduction) which form a cycle. The basic furctions of the sub-alroritnms are as follows:
(a) heduction breaks up the query into irreducible comporerits ard puts them ir a certair sequeritial order.
(b) Subquery Sequersirf uses the result of Reduction ard rerierates in successior subqueries each of which cortains a sinfle irreducible comporent torether with ore-variable clauses. As each subquery is gerierated it is passed to Tuple-Substitutior, ard the rereration of the rext subquery awaits return of the
result.
(c) Tuple Substitutior mariages the process of sub-

- stitutirg tuple values. It calls Variable-Selectior to select a single variable for substitution. After substítutirig eacn tuple for that variable, it passes the resulting reduced query to Reduction ard awaits the returr before substitutirg the rext value.
(d) Variable Selection is where rost of the optimi-
, zatiori takes place. It estimates the relative cost of substituting for each variable ard chooses the variable with the mirimum estimated cost. In so do*irr, it may nave to preprocess some ore-variable subqueries.
$\cdots$
The details of the sub-algoritnms wis be described ir the rext few sections.

4. Reduction Algorithm

The input consists of a multivariable query Q, and the output consists of the irreducible components of $Q$ arranged in an àppopriate sequential order. This sequence is passed to Subquery Sequencing, and the result relakion for $Q$ is returned. The basic steps of the argorythm are illus. trated below.


## Figure 4.1 Reduction Algorithm

Let $X=\left(X_{1} ; X_{2}, \ldots, X_{n}\right)$ denote the variables of $Q$ and let $T(X)$ aff $B(X)$ denote its target list and qualification respectively. We assume that $B(X)$ is expressed in conjunctive normal form

$$
B(x)=\bigwedge_{i} C_{i}(x)
$$

Where each clause $C_{f}(X)$ contains only disjunctions. Now consider a binary ( 0 or 1) matrix with $p+1$ rows corresponding to $T(X)$ and the $p$ clauses, and with $n$ colums corresponding to the variables $X_{1}, \ldots, X_{n}$. An ertry of 1 will denote the presence of a variable ir a clause (or target list), and 0 will denote its absence. We shall call this the incidence matrix. For example 3.1 this matrix. is given by
 steps for which detailed algorithms remain to be provided. First, we need a test for conriectedness, and to separate $Q$ into disjoint components if it is not connected. Second, we need an. algorithm to separate a connected query into irreducible comporierts and to put them in a suitable sequential order.
(a) Corirectivity Algorithm


Figure 4. 2 Cornectivity
If the corirectivity alporithm results in a matrix with a sirgle row which is not all 1 's then the variables corresponding to the zero-ertries are superfluous and car be elimirated. If the firal matrix has more thar one row, then the sets of variables corresponding to different rows must be disjoint. If we keep track of the original rows which are combined to make up each of the rows of the fi-
ral matrix, then the connected comporints of the query cara be separated.

Corisider example 3.1 , modified by the deletion of C4. The irciderce matrix row has the form

|  | $S$ | $P$ | $Y$ |
| :---: | :---: | :---: | :---: |
| $T$ | 1 | 0 | 0 |
| C1 | 1 | 0 | 0 |
| C2 | 0 | 1 | 0 |
| C3 | 0 | 1 | 0 |
| C5 | 0 | 1 | 1 |
| C6 | 0 | 0 | 1 |

Applyirg the correctivity algorithm, we get successively

|  | S | P | Y |
| :---: | :---: | :---: | :---: |
| $\mathrm{T}, \mathrm{C} 1$ | $\cdot 1$ | 0 | 0 |
| C 2 | 0 | 1 | 0 |
| C 3 | 0 | 1 | 0 |
| C 5 | 0 | 1 | 1 |
| C 6 | 0 | 0 | 1 |


|  | $S$ | $P$ | $Y$ |
| :---: | :---: | :---: | :---: |
| $\mathrm{~T}, \mathrm{C} 1$ | 1 | 0 | 0 |
| $\mathrm{C}, \mathrm{C} 3, \mathrm{C} 5$ | 0 | 1 | 1 |
| C 6 | 0 | 0 | 1 |



Herice, the query is rot cornected ard the coriraected comporierts are ( $T, C 1$ ) and (C2,C3,C5,C6).
(b) Reduction irito Irreducible Comporients

Let $Q$ be a corracted multivariable query. We observe that it is reducible if the elimination of any ore variable results ir $Q$ beirg disconrected. Let a variable with this property be called a joinirg-variable. Thus, $Q$ is irreducible if and only if nore of its variables is a joindeg-variable. Joirifig-variables have some important properties which greatly facilitate the reduction algorithm, arid these are summarized as follows:

- Proposition 4.1 Suppose that $X$ is a joiringvariable of $Q$ such that its removal discorirects $Q$ irito $k$ corarected comporients. Ther ary joiringvariable of orie of the componerts is a joinirgvariable of $Q$, and every joiring-variable of $Q$ is a joining-variable of one of the comporients. Further, successive elimiration of two joining variables ir either order results in reducirig $Q$ to the same disjoint comporierits.
proof: Each joiririg-variable joirs ,a rumber of comporests which car overlap orily or the joiningvariable. Let $X$ be a joiring-variable of $Q$ which joins componerits $Q_{1}, Q_{2}, \ldots, Q_{5}$ Let $Y$ be a joinirg variable of ore of these compolerts, say $Q_{1}$. Ther, Y. joirs comporierts $Q_{11}, Q_{12}, \ldots, Q_{j}$ of $Q_{1}$, orily one ( $Q, 2, \ldots, Q_{1}$ ) overlaps the remairder of $Q$ only or $Y$ and ${ }^{\prime}$ is a joiririg-variable of $Q$. Corversely, let $Y$ be a joiririg-variable of $Q$, ard joir comporierits $Q_{1}, ;$
 $\left.Q_{2}, \ldots, Q_{k}\right\}$, car coritair $\underbrace{}_{1}$, say $Q_{1}$. Ther $\left\{Q_{2}, \ldots, Q_{j}\right\}$ ard $\left\{Q_{2}, \ldots, Q_{k}\right\}$ must be disjoint since each $Q_{i}{ }^{\prime} i^{j} \geq 2$, can overlap its remairider iry $Q$ only on . $X^{1}$ and, rone of $\left\{Q_{2}{ }^{\prime}, \ldots, Q_{j}{ }^{\prime}\right\}$ coritairis $X_{\text {, }}$ Herice., $C_{2}{ }^{\prime}, \ldots, Q_{\text {in }}$ are subsets of $Q Q_{j} j$ is ined to it orily by $V_{\text {, }}$ so that $Y$ is a joiriry-variable of $Q_{1}$. It is clear that $Q$ nas comporients $\left\{Q_{2}, Q_{3}, \ldots, Q_{k}\right\}$ each joined by only $X,\left\{Q_{2}^{\prime}, Q_{3}, \ldots,_{j}^{\prime}\right\}$ 3eacn joined by
cray $Y$; and a compcrent $\Omega$.joired by bctn $X$ and $Y$. Lliniration of $X$ ard $y^{x y}$ either cirdén results lir,

 ard $Y$ remciod:

The siubstarce of Propositior: 4.1 is illus-


Figure 4.3: Joiring-Variables

The results of Propositicn 4.1 mean that we car firid the irreduchble comporerts of 0 by successively checkirig each variable for the possibility of beirig a joining variabale. Each variable orily reeds to be exahined orice, ard the order they are tested is immaterial. Further, since a variable is joiring if and orly if its elimination discornects $Q$, we car use the connectivity algorithm for the test.

Take the irciderice matrix of $Q$ ard eliminate from it all rows with orily a single "1". . Begirinirig , with the first, elimirate each column in turn ard test for corarectedress. Suppose that wheri columra $m$ is elimirated Q breaks. up irto $k$ cornected comporierts with $r_{1}, r_{2}, \ldots, n_{k}$ variables respectivëly. Ther, these correspord to comporients of $Q$ with $r_{1}+1$, $r_{2}+1, \ldots, r_{r}+1$ variables respectively, ariy pair of whicn overlap orily on $X$. We car now : proceed to test columns $m+1, \ldots$, , We note that each of the variables $X_{m+1}, \ldots, X_{\text {a }}$ occur in orily ore of the comparrerts so ${ }^{\text {m+ }}$ that after the moth column (i.e.) the first joining-variable) the tests are performed on matrices of reduced size.

Each irreducible comporient of $Q$ corresponds to one or more row of the incidence matrix, ard can be represerited by the "logical or" of the corresponding rows. Hence, Qucar be represented in terms of its irreducible comporerits by a matrix with variables as columns and comporients as rows. We shall call this the reduced-incidence-matrix. It is conveniert to arrarige the rows astfollows:
(1) One-variable rows except the target list. $/$ (2) Componerts which are one-overlapping after deletion of ore-variable clauses ard which do rot contair the target iist. These should be grouped accordirg to the joiring variable.
(3) Other comporients which do not contain the target list.
(4), The component which coritains the target list.

For example 3.1 the resulting reduced irciderice matrix is given by:

|  | $S$ | $P$ | $Y$ |
| :---: | :--- | :--- | :--- |
| $\overline{C 1}$ | 1 | 0 | 0 |
| $\bar{C} 2$ | 0 | 1 | 0 |
| $\bar{C} 3$ | 0 | 1 | 0 |
| $\bar{C} 6$ | 0 | 0 | 1 |
| $\overline{C 5}$ | 0 | 1 | 1 |
| $T, C 4$ | 1 | 0 | 1 |

8. Woptimate of Result Parameters"

Ir order to use $(7.5)$ ir $(7.4)$, we must krow the number of pages occupied by the range relations for every $q_{\alpha}$ in the sequence $S_{0}$. Werrote that $S_{i}$ is a sequerice ard rot a set, so that the rarige relatior of a query may involve the result relations of queries which precede it. Therefore, knowing the sizes of the rarise relations of $Q$ is not sufficient. to determine (7.5) for the $q_{\alpha}$ 's. Since we don "t. warit to execute the sequerice $S_{f}$ except for the optimal i, we must rely on a procedure to estimatie the sizes and other parameters of the result relation for a query.

Corisider a query $Q$ with rarge relatiors $R_{1}, R_{2}, \because, R_{r,}$ a target list $T(X)$ ard'a qualification $B(X)$ Let the domairs of $R_{i}$ be deroted by $D_{j}$, $j=1,2, \ldots, d_{i}$. Each $R_{1}$ is by defirition a subset $\begin{aligned} & \text { ot. }\end{aligned}$ $j \leq d_{i}{ }^{\circ}$
Herice, the product. $T R_{i}$ is a subset of

$$
\begin{align*}
& D=\prod_{i \leq n} j \leq d_{i} \tag{8.1}
\end{align*}
$$

To determine what subset. of $T T R_{i}$-satisfies $B(X)=1$ requires accesses to the actual relations, but to determire what subset of $D$ satisfies $B(X)=1$ requires only krowing the domains $\left\{D_{j j}\right\}$. The storage required to represert $\left\{D_{i j}\right\}$ is ifin $^{f}$ gereral far: less thar that required for $\left\{R_{i}\right\}$.

Let $R(Q)$ derote the result relation "of 0 . We car estimate the cardirality of $R(Q)$ as $(8,2),|R(Q)|=\left|\prod_{1 \leq r} \dot{R}_{1}\right| \cdot\{$ fraction of $D$ satisfying $B(X)=1\}$

The domains of $R(Q)$ can be estimated by evaluatinp $T(X)$ or the subset of $D$ which satisfied $B(X)=1$. That is, the kth domair of $R(Q)$ is estimated to be
(8.3). $\quad\left\{T_{k}(X) ; X \in D, B(X)=1\right\}$

Ir most cases $D_{i j}$ has sufficient regularity to permit it to be represented by just a few parameters. For example, $D_{i}$ might be simply all integers between a ard $b$. Thus, the storage requirement for keeping track of the domains for the result. relatifons of the sequerice $S_{i}$ can be expected to be reasorable.

## 6. Tuple Substitution

The ir put to tuple substitution is a query $Q$ consisting of a single irreducible component in variables $X_{1}, X_{2}, \ldots, X_{1}$, zero or more one-variable clauses in each of the variables, and the range reCations $R_{1}, R_{2}, \ldots, R_{p}$ of the variables. It returns the result relation Co the calling process. $^{\text {c }}$

The first thing that Tuple Substitution does is to call Variable Selection which takes $Q$ and the range relations and chooses a variable to be substitoted for. In order to make this choice it may have to process some or all of the ore-variable clauses to restrict the ranges. Thus, in general, it returns $\left\{Q^{\prime}, R_{2}, R_{n}, \ldots, R_{n}^{\prime}\right\}$ ard the variable to be substituted for (say X $)$ for each tuple in R
Q becomes a $(n-1)$-variable query $Q(\alpha)$ in

 variables $X$, $X_{2} \ldots, X^{\circ}$ For each $\alpha$, $Q(\alpha)$ is results. to $Q^{\prime}\left(\alpha^{\prime}\right)$ for all $\alpha$ in $R_{r}$ are accumulated and returned to the calling process.

```
i
```


## 7. Variable Selecticr

This is the neart of optimizatior. The input is a multivariable query which is irreducible except for ore-variable clauses. As its rame surpests, the task of this portion of the deccmpositior. alecrithn is to select a variable for substitutior, dIthousn to do so it may alsc have to process some cf the cre-variable clauses.

Corsider a query o with variables $\because$
 becomes an (r.-1)-Nariable query $\ell_{i}(\alpha)$. It is likely that $Q_{i}(\alpha)$ takes the same amcurt of time tc process fer every $\alpha$, ard in most instarces every $\alpha$ in $F_{i}$ has to be used. !lerice,
(7:1) Cost of processirer 0 if $X_{i}$ is substituted
$=$ (cardirality cf $F_{i}$ ) x Cost of processirir $O_{i}$
The first thourht, therefore, is to chocse $\gamma_{i}$ with the smallest rarge. However, this is not entimal for several reascris.

Pirst, it may be possible to reduce some cor all of the relaticrs $R_{1}, \pi_{2}, \ldots, f$, by oreprocessire cre variable clauses. Sncald this be dore for all, for: some, on for ricre of the variables? If all of
 volves $2^{\text {n }}$ choices. The situatior is furtner conplicated by the fact that for a fiver: variable the decisicn as to whether to preprocess. the onevariable clauses deperds er whetner the variable is choser for substitutior. If it will be ohoser fo. substituticr ther its rarse should be reduced as much as pcssible. If not, preprocessing may be a waste of time. Or: the cther hard which variable should be choser deperds r.ct so much or $R$ as or the reduced $R_{i}$ Let $\quad$ ( $X_{i}$ ) deracte the ore-variable
 rarse after $\overline{0}\left(X_{i}\right)$ is pocessed. The followir.f policies seem to be reasorable alterratives:
(a) Preprocess every $\tilde{Q}\left(X_{i}\right)$, basirir the policy or the arrumert that the ccist of processir. ore-variable queries is relatively small ard it is importart to choose the variable for substitiuticr vell.
(b) " Or the basis of $\tilde{O}\left(X_{f}\right)$, a decisior is made for each variable whether to preprocess or rot. Variable selecticr takes place after prepro-

The version of INGRES completed in Jaruary, 1976, opts for policy (a). In part, it is because in this version the variable selection is then based sclely cre the cardiralities of the reduced ranges and ro other information. It is important, therefore, for these cardiralities to be accurate.

For (b) a workable policy is to use $\tilde{Q}\left(X_{f}\right)$ to estimate the size of R $^{\prime}$, for each 1 , and preprocess orily if $X_{j}$ is likely tol $^{2}$ be a contender for selectior. For example, we might choose the top three. corteriders for preprocessing, or preprocess every variable for which the estimated size of $R_{1}{ }^{\prime}$ is less thar mir $\|_{\mathrm{j}} \mathrm{l}$. Ore good feature of (b) is that except for $\forall$ ery uriusual situatiors, the actual variable selected will be amort those. which have beer preprocessed, and $n_{0} 0$ further processing is necessary before substitutior.

A second ard more importarit objection to the strategy of choosirg $X_{i}$ with the smallest ratree is that the complexity of $Q_{i}$ car vary greatly with $i$ ard this must be taker Into accourt in ary strategy which lays claim to being even near-optimal. What must be determired is the extert to which $Q$ car. be reduced as consequerce of substituting for $X_{i}$..

Assume that we choose either (a) or (b) for the policy on preprocessing one-variable clauses so that that decision is decoupled from the selection of variable. Ne car assume that the query at this poift coksists of a single irreducible component with some ore-variable clauses. The crux of the matter is how the irreducible componert is affected by the substitution. Assume that whatever preprocesslag is to be dore has beer. dorie. Let the query be derioted by $Q$ Let $X_{1}, X_{2}, \ldots, X_{n}$, be the vari-. ables, ard let $R, \cdots R_{2}, \ldots, R_{r}$ be their ranges. ${ }^{n}$ Let $Q_{i}(\alpha)$ derote the nesulting query from substituting $\alpha$ for $X_{i}$ ir $Q$. Le't $\dot{C}(Q)$ derote the minimum cost of processifg Q. Ther
(7.2) $C(Q)=\min _{i}\left\{\sum_{\bar{R}_{i}} c\left(Q_{i}(\alpha)\right)\right\}$
'where $\bar{R}_{i}$ deriotes .the set of tuple-values which have to be substituted for $X_{1}$. In most instances this is simply $R_{i}$, althcugh as He irdicated earlier there are exceptions.

Equatior (7.2).is a dyramic programming
equation for the optimization problem at hard. As it stands, it is not too useful; since how $C(Q)$ depends on $Q$ is rot $k r_{1} \mathrm{wr}_{2}$. However, (7.2) $\because$ is a suitable starting poiret for optimization. The sariable selected will corresporid to the value of $i$ which minimizes an estimated value for
(7.3)

$$
C_{i}=\sum_{\alpha \in \widetilde{T}_{i}} C\left(Q_{i}(\alpha)\right)
$$

Although we have in effect transferred the optimizaton problem to one of estimating cost; the latter is amenable to a variety of heuristic approaches Consider some of these:
(ii) Suppose we take the estimate ". of $C\left(Q_{j}(\alpha)\right)$ to be independent of $\alpha$ ard 1 . Then ; the minimum $C_{i}$ corresponds to the smallest $R_{i}$. This somewhat ${ }^{1}$ simplistic policy is what has beer implemerited in the version of INGRES operational as of Jariuary, 1976.
(ii) We observe that unlike $Q, Q_{i}(\alpha)$ is rot irreducible. Ore should therefore call Reductior-Subquery-Sequericirg to reduce $Q_{i}(\alpha)$ to ... a sequence. $S_{j}$ of subqueries; each of which is irreducible except for ore-variable clauses. Now, $\alpha$ enters the subqueries orly as a parameter, and the sequence $S_{i}$ is really independent of $\alpha$. Thus, we have

$$
\begin{equation*}
c\left(Q_{i}(\alpha)\right)=\sum_{q_{\alpha} \in S_{1}} c\left(q_{\alpha}\right) \tag{7.4}
\end{equation*}
$$

Since the structure of $Q_{1}(\alpha)$ has now beer represented, we can accept a relatively crude estmate for $C\left(q_{\alpha}\right)$. For example, we might take the estimate of $C\left(q_{\alpha}\right)$ to be

$$
\begin{equation*}
C\left(q_{\alpha}\right)=\because T_{j} P\left(R_{f}\right) \tag{7.5}
\end{equation*}
$$

where $f_{f}$ are the ranges of $q$ ara" $P(R)$ is the number of pages occupied by *R. trained from using. (7.4) ir $\mathrm{ir}_{2}(7.2)$

$$
\begin{equation*}
C(0)=\min _{1}\left\{\sum_{\alpha \in \bar{R}_{1}} \sum_{q \in S_{1}} C\left(q_{\alpha}\right)\right\} \tag{7.6}
\end{equation*}
$$

This is truly recursive, since $Q$ and $q \alpha$ are queries of tho same restricted type (viz, irreducible except
for one-variable clauses). If the rumber of variables in : $Q$ is rot erormous (in practice, very few queries contain more than 4 or 5 variables) one might try to pust the recursion (7,6) all, the way down to one-variable queries, but using small samples for the range relations of Q.. It is:very likely that the costs of different paths in the decisiop tree vary widely, and only a few are contenders for the optimal path. With efficiert maragemert, this approach need not be prohibitively expensive.

These are but three possidie approaches to estimating : $C(Q)$. other approaches iricluding some variants and combinations of these are under consideratior. We expect to implemerit at least the three outlined above for experimertal evaluation. Indeed, (i) has been implemented, "and (if) is 'in the process of beire fimplemented.
8. "Nostimate of Result Parameters"

In order to use (7.5) ir (7.4) we must krow the rumber of pages occupied by the rarge relations Cor every ' $q_{\alpha}$ in the sequence $S$. We rote that" $S_{i}$ ' is a sequerice ard not a set, so that the rarge relation of a query may involve the result relations of queries which precede it. Therefore, krowing the sizes of the ranse relations of $Q$, is not sufficient to determirie (7.5) for the $9_{\alpha}$ s. Sirice we don't. wart to execute the sequerce $S_{\text {p }}$ except for the optimal i, we must rely on a procedure to estimatie the sizes and other parameters of the result relation for a query.

Corsider a query $Q$ with rarge relatiors $R_{1}, \dot{R}_{2}, \therefore \ldots, R_{r}$, a target inst $T(X)$ and a qualification $B(X)$ Let Ehe domairs of $R_{i}$ be deroted by $D_{i j}$, $j=1,2, \ldots, d_{i}$ Each $R_{i}$ is by definition a subset $\bar{j} j^{\prime}$. $j \leq d$.
Herce, the product. $T R_{i}$ is a subset of

$$
\begin{equation*}
D=T \prod_{i \leq n} \prod_{j \leq d} D_{i j} \tag{8.1}
\end{equation*}
$$

To determine what subset. of $\Pi R_{i}$-satisfies $B(X)=1$ requires accesses to the actual relations, but to determine what subset of $D$ satisfies $B(X)=1$ requires only knowing the domairis \{D,j\}. The storage required to represert $\left\{D_{i j}\right\}$ is. in gerseral far. less thar that required for $\left\{R_{i}\right\}$.

Let $R(Q)$ derote the result relation of 0 . We car estimate the cardinality of $R(Q)$ as (8.2) $|R(Q)|=\left|\prod_{i \leq n_{1}}\right| \cdot\{f r a c t i o n ~ o f ~ D$ satisfying $B(X)=1\}$

The domains of $R(Q)$ can be estimated by evaluating $T(X)$ or the subset of $D$ which satisfied $B(X)=1$. That is, the kth domair of $R(Q)$ is estimated to be

$$
\begin{equation*}
\left\{T_{k}(X) ; X \in D, B(X)=1\right\} \tag{8,3}
\end{equation*}
$$

In most cases $D_{1}$. has sufficient. regularity to permit it to be represterted by just a few parameters. For example, $D_{i j}$ might be simply all integers between a and $0 . \therefore$ Thus, the storage requirement for keepirg track of the, domains for the result. relatifors of the sequerice $S_{i}$ can be expected to be reasonable.

Since the sizes of the tuples are always krown, the rumber of pages required for each of the result relatioris for the sequence can be computed from the estimated (8.2), which in turr is computed fron the estimated domains using (8.3).

## 9. Summary

In this paper we have presented a. detailed account of how multivariable queries are decomposed in system INGRES. The basic ingredients of the decomposition are two in number:
(a) To discover pieces of a query which are joined to the remainder by a single joiningvariable.
(b) To substitute for a variable.

The overall strategy is to break up a query at the joining-variables whenever this is possible, and to: select a variable for substitution which incurs a "minimum cost" whenever substitution can no longer be postponed. A detailed algorithm for reducing a query. into irreducible components has been given: Alternative approaches to estimating costs have also been discussed.

Optimization itself incurs a cost which has not been taker into consideration: For simple queries; elaborate optimization may well do more Charm than good. The approach to resolving this difFifulty that we have opted is one suggested by M.R. Stonebraker. Suppose that we have two or more strategies st ${ }^{\text {, st }}, \ldots$, st $t_{n}$, each one being better than the previous one but also requiring a greater overhead. Suppose we begin a query on st, arid run it for an amount of time equal to a fraction of the estimated overhead of st. . At the end of that time, by simply counting the number. of tuples of the first substitution variables which have already been processed, we can get an estimate for the total processing time using st. If this is significantly greater than the overhead of st, then we switch to st ${ }_{1}$." Otherwise we stay and complete processing the query using st obviously, the procedure can be repeated on st, , to call st ${ }_{2}$ if necessary,, and so forth. st may correspond, ${ }^{2}$ for example, to progressively more levels in the decision tree, or to progressively more elaborate estimates of result arameters, or better sampling.

We have not addressed the question of optimeizing the processing of one-variable queries. Some optimization is currently being done in INGRES, and this is described elsewhere [STON76].

In the appendix we have given a brief description of how INGRES is implemented. The ortginal design of the implementation was primarily the

WORK of M. N. Storiebraker and G.D. Held. Redesigr of process 3 , qud ir particular the design of the oquery tree ard the implementation of the decomposition alEorithm in the current verision (as of Jariuary, 1976) have beer largely the work of Peter Kreps. We have alsc ircluded is the apperdix specificatiors of the priricipal data structures reeded for our decompositiots algorithm.

Ore of us (E.W.) is resporsible for introducirg the conceptual framework in which the decompositior algorithm rests, viz. ther policy of trarisformirg a multivariable query to ore dimersion al ores, and the strategy of alternating between. reduction ard tuple substitution. We have collaborated or the reduction algorithm, ard or the heuristics for variable selection. "The implemertation of the full algorithm as well as moritoring. subsystems for the performance evaluation is being desigred ard executed by K.A.Y: The decompositior alcorithm; being at the neart of INGRES, has enjoyed the attertion of mary participarts of the project. It is difficult to remember, who suggested what, but the three aforemertioned colleagues have all made importart contribytions. Ir particular, as in every aspect of INGRES, the infiuerce of M.R.S. is discerrible throughout our algorithma
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## APPENDIX A.

## System Organization

INGRES, Interactive Graphics and Retrieval System, runs on a PDP $11 / 45$ under the UNIX operating system[RETC73]. The entire system is written in the programming lariguage ${ }^{\prime \prime} C^{"}$ [RITC74]. It has four ma= jor components which are orgarized as shown below.


These four components are set up às processes under UNIX and communicate through the usej of pipes. The user interface can be one of several forms: an interactive text editor, a graphics interface [MCD074], an interactive English-like language [CODD74], or part of a host programming language [ALLM75]. The parser accepts the user's query and processes it into a tree in corijunctive normal form. This query tree and a table of relations declared in the : RANGE statements are passed to decomposition. The decomposition process contains not only the decomposition algorithm but also the one-variable. query processor. The utilities process contains many functions which can be used by the system or the user.

## Data Structures

There are three main data structures which are used duritrg decomposittor of a-query.

Rarre Table:
Some of the information for this structure is sathered durirg parsing añ passed to decompositior as an crderedmatrix. It is then put intc a matrix, each ertry of which has the following form:


The parser serds a table of relation rames which have beer declared ir RANGE statemerts; the order of these inames irdicate the variable associated with each. These are relid. The secorid entry is a pciriter to ar in-core copy of the system catalogue descriptior for that relation. The third entry is a flag which is set wher the correspording vaniable, has beer selected for substititior.

The use of this table will aid decompositior irs the use of temporary relations. Wher a rev rarige is created for a. variable by execution of of onevariable query, the ertry in the rarige table for that ertry is the same, except for the pointer to the catalorue description. The relid is always the orifiral relation name for that variable and the descriptor is for the current subrelatior it is rariging over...Ir this way, if a temporary relation must be created several times during the process of substituticri, the same temporary relation rame and descriptor car be reused by simply deletirg the old tuples from the previous fteration: This saves much overhead fri the creation of temporany relations.

Ircoiderce Matrix:
This is a birary matrix of clauses (or supqueries) vs variables which is used withir decompositich to represent the currert queny" under corisideration $\therefore$ It is used during. reduction to determire all irreducible subqueries and can be used durirg selecticr to represent the component subqueries in a compact form. This matrix will also cortair ar, ertry for each clause which points to the actual clause so that it may be easily obtained when it is recessary to build a query tree for execation of a subquery.


#### Abstract

Query Tree: The parser sends a list representing the query tree to decomposition whichcthen rebuilds the query tree adding useful information as it is reconrived. The general form of this tree is a root node with the target list of the query as the left branch ard the qualification as the right branch: Since the query is in corijurictive normal form, all the interinediate nodes along the right side-will be AND (conjurictiori) nodes.



where left and right are the pointers to the respecfive branches. The second entry defines the structore within the rode ard this varies depending on the type of rode:

For nodes representing arithmetic operators, disjurictions (OR), result domains and constants, the structure $1 s:$
struct symbol
$\left\{\begin{array}{c}\text { char type } \\ \because \quad \text { int value[] }\end{array}\right.$
\}
where type is a code representing the type of the 4 rode (i.e., plus, mirius, OR, etc.) and len is the length in bytes of value. value is a variable f length field. (0-255 bytes) and goritains the appropriate value for that type of node. For example, if the rode is representing: a constant then the ह. value coritairis the actual constant.

For rodes representing variableattribute E. SALARY the structure is:

```
struct symbol
{ : char type;
    char ler;'
    char, varro, attro;
    char frmt, frml;
    char *valptr;
}
```

where type is the same as above arid len is fixed. varro is an index into the range table for the correct variable; attro is the domair number (from the system catalogue) of the correct domalin referenced. frimt arid frml give the format of the attribute (i.e.; Ab, I2; etc.). This is used to determine new domain types and for calculations. The last ertry is used durirg tuple substituticr. If a particular variable is selected for substitution, all variable.attribute riodes irvolvirg that variable will become nodes represeritirg coristarts. But the tree itself reed not be changed. This field, valptr, is simply set to point to the coristart yalue that should be used. This position remains fixed so wher a new tuple value is substituted, the pointer does rot charge, orily the value it is poirting to changes. Ir this way, a rew tree is not $r_{2} e e^{2}$ for each level of substitution or for each iteration of substituticr values. If the pointer is. zero, the variable information is used; if it is rorizero, it is a constare riode.

For ricdes representing the root or conjurctions (AND), the structure is:

```
struct symbal
{ : char type;
    char len;
    char tvare;
    char lvarc;
    irt lvarm;
    intt rvarm;
```

where type is the same and len is fixed... tvarc ard 1varc are both courts of the variables used. tvare is the number of variables in the sub-tree below tnis" rode and lvarc is the number of variabies in the left branch. Sc for the root rode, tvare the the tetal number of variables in the query ard lvaro is the rumber of variables in the target list. For an AlvD riode, tyars is the rumber of yariables in the remairing. clauses and lvare is the number: of vari-
ables ir the single ciause of its left branch. lyanm and rvarmare bjt maps of the variables used Ir the left and right branches of the node respectively.

Tnis structure is rot as costiy as it might appear. It is true that during decomposition many subqueries are created and executed many times, but it should be roted that all of these subqueries use clauses which appear in the original query. The target lists may charge, but no rew clauses are ever created except through substitution Since this is true, wher a subquery is to be executed; a query tree car be constructed using rodes from the original tree. A, rew root rode must be created for eachlsubquery and for some target list nodes, but all the" AND rodes car simply be detached from the origiral query tree ard added to the new query tree.

$\underset{ }{4}$


